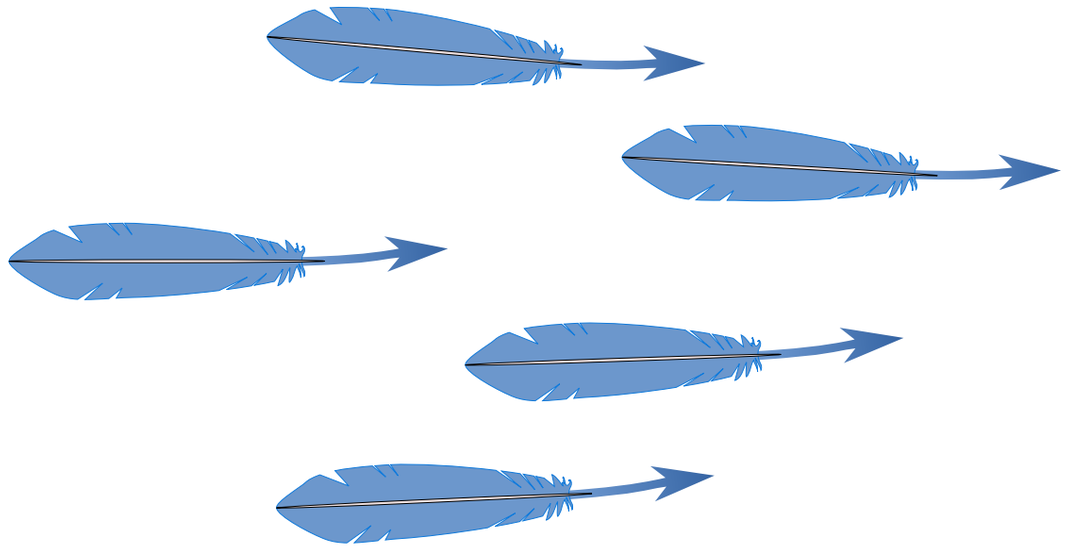
EIAS and numerical math — introducing VecTcl



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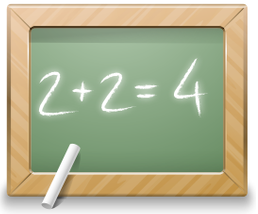
EuroTcl 2014

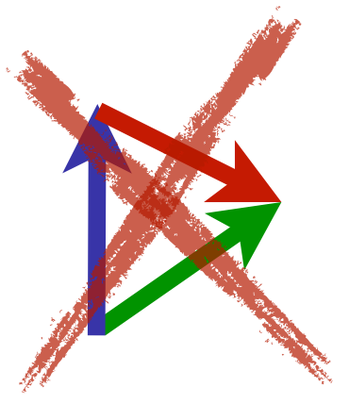
What is numerical / vector math?



Tcl has (scalar) math in the core:

$$x = \frac{1}{2a}\left(b \pm \sqrt{b^2 - 4ac}\right)$$

```
set x [expr {($b+sqrt($b**2-4*$a*$c))/(2*$a)}]
```



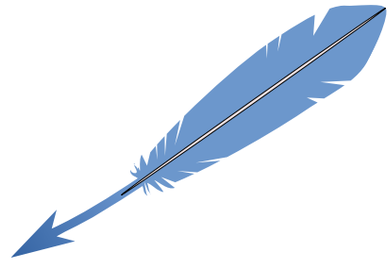
There is no direct support for vector math:

$$x = \vec{a} \cdot \vec{b} = \sum_i a_i b_i$$

```
set x 0.0
foreach ai $a bi $b {
  set x [expr {$x+$a*$b}]
}
```

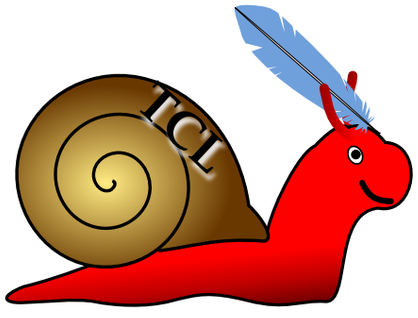
Tcl is more complicated / uglier than textbook math

VecTcl demo / Why do we need it?



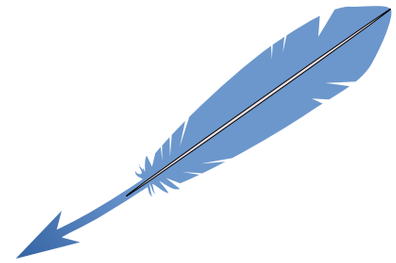
Cool applications: Data fitting, image processing, Physical modelling, 3D graphics

Python has it ;)



Tcl is often too slow

Linear regression



Math

$$
\begin{aligned}
\bar{x} &= \frac{1}{N}\sum x_i \\
\bar{y} &= \frac{1}{N}\sum y_i \\
\beta &= \frac{\sum_i (x_i - \bar{x}) \cdot (y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2} \\
\alpha &= \bar{y} - \beta\bar{x}
\end{aligned}
$$

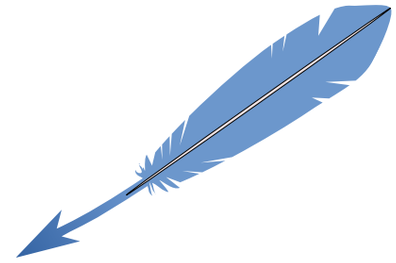
Tcl

```
set xsum 0.0; set ysum 0.0
foreach x $xv y $yv {
    set xsum [expr {$xsum + $x}]
    set ysum [expr {$ysum + $y}]
}
set xm [expr {$xsum/[llength $xv]}]
set ym [expr {$ysum/[llength $xv]}]
set xsum 0.0; set ysum 0.0
foreach x $xv y $yv {
    set dx [expr {$x - $xm}]
    set dy [expr {$y - $ym}]
    set xsum [expr {$xsum + $dx * $dy}]
    set ysum [expr {$ysum + $dx * $dx}]
}
set b [expr {$xsum / $ysum}]
set a [expr {$ym - $b * $xm}]
```

VecTcl

```
vexpr {
   xm=mean(xv)
   ym=mean(yv)
   beta=sum((xv-xm).*(yv-ym))./sum((xv-xm).^2)
   alpha=ym-beta*xm
}
```

Linear Least Squares



Even easier: the matrix way

$$A = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_N \end{pmatrix} \quad \Rightarrow \quad \text{solve } Ap = y \quad \Rightarrow \quad p = \begin{pmatrix} \alpha & \beta \end{pmatrix}$$

VecTcl

```
vexpr {
    A=hstack(1, xv)
    alpha, beta = A \ yv
}
```

Linear Least Squares

Even easier: the matrix way

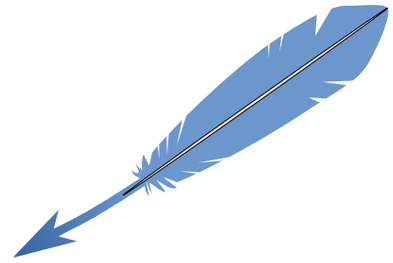
$$A = \begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_N & x_N^2 \end{pmatrix} \quad \Rightarrow \quad \text{solve } Ap = y \quad \Rightarrow \quad p = \begin{pmatrix} \alpha & \beta & \gamma \end{pmatrix}$$

VecTcl

```
vexpr {
    A=hstack(1, xv, xv.^2)
    alpha, beta, gamma = A \ yv
}
```

no way to do it in pure Tcl
(short of writing LLS matrix decomposition)

Why a new package?



Q: There is TIP 363 , TIP 420, and a couple of extensions. Who needs a new vector package?

A: VecTcl was designed to be

- Easy to use
- Interoperable
- General
- Easy to build and run
- Fast and memory efficient

VecTcl beats all competitors in all of these points

A word about the TIPs...

TIP #363: Vector Math in the Tcl Core

```
set ListA {1 2 3}
set b "$ListA";# -> "{1 2 3}"
set b "{*}$ListA";# -> "1 2 3"
```

*„The proposed approach will enable **eventual** incorporation of vector-math to the Tcl engine“*

```
expr $ListA + {4 5 6};# -> "{5 7 9}"
```

Q: Why on earth break everything to enable „eventual“ incorporation of vector-math to the Tcl engine?

```
set name "Mr. John Ousterhout"
puts "Hello, $name"
# Hello, {Mr. John Ousterhout}
```

breaks „Hello world!“

A word about the TIPs...

TIP #363: Vector Math in the Tcl Core

```
set ListA {1 2 3}
set b "$ListA";# -> "{1 2 3}"
set b "{*}$ListA";# -> "1 2 3"
```

*„The proposed approach will enable **eventual** incorporation of vector-math to the Tcl engine“*

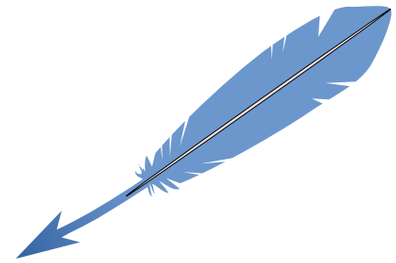
```
expr $ListA + {4 5 6};# -> "{5 7 9}"
```

Q: Why on earth break everything to enable „eventual“ incorporation of vector-math to the Tcl engine?

```
set ListA {1 2 3}
vexpr {ListA + {4 5 6}}
```

works and doesn‘t break anything

A word about the TIPs…



TIP #420: Vector Math in the Tcl Core

```
set a {1 2 3}
set b {4 5 6}
vexpr $a $b + 2 *
# {10.0 14.0 18.0}
```

It actually works — there is an implementation for 3D

*„vexpr is a vector expression parser. It operates using **reverse-polish notation** [...] Why? Well **mostly for ease of implementation**. Partly because there is no PEMDAS equivalent order of operation for matrices and vectors.“*

Q: Only 3D? No operator precedence for ease of implementation? Use a real parser generator!

```
set a {1 2 3}
set b {4 5 6}
vexpr {2*(a+b)}
```

Textbook-compatible operator precedence in N-D

Ease of use

VecTcl

```
vexpr {
   xm=mean(xv)
   ym=mean(yv)
   beta=sum((xv-xm).*(yv-ym))./sum((xv-xm).^2)
   alpha=ym-beta*xm
}
```

Input/Output: just use
`xv, yv, alpha, beta`

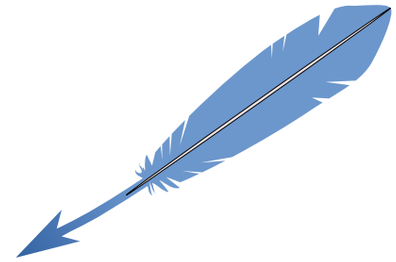
BLT / rbc

```
vector create x
vector create y
vector create alpha
vector create beta
x set $xv
y set $yv
beta expr {(mean(x*y)-mean(x)*mean(y))/(mean(x^2)-mean(x)^2)}
alpha expr {mean(y)-beta*mean(x)}
set alpha [alpha index 0]
set beta [beta index 0]
```

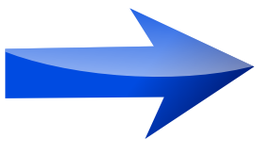
Input from Tcl

Output to Tcl

Interoperability

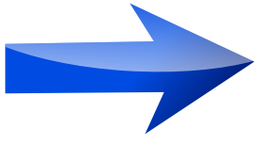


- Tcl has a type for sequences of numbers: lists
- VecTcl operates on lists (and lists-of-lists)
- VecTcl uses Tcl variables and commands (functions)



Simply use your variable in pure Tcl, VecTcl, math::linearalgebra, odielib, lapack...

```
package require math::linearalgebra
namespace import math::linearalgebra::*
set X [mkDingdong 3]
vexpr { X \ { 1 2 3} }
```

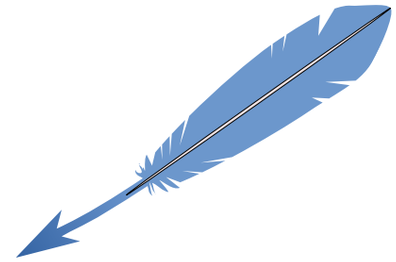


Simply use other commands in VecTcl

```
vexpr { orthonormalizeColumns(X) }
```

Different from NAP, BLT, tcl-tna, narray, tensor... but like the TIPs

Generality



VecTcl supports:

- integer, floating point and complex numbers
- scalars, vectors, matrices and N-rank tensors
- string rep carefully designed to comply with lists

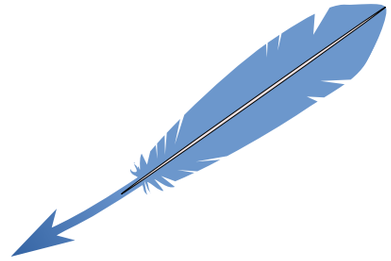
```
set x {1 2 3} ;# an integer vector of length 3

set y {{1.0 3.0} {3.0 5.0}} ;# a 2x2 floating-point matrix
set y {{1.0 3.0 5.0}} ;# a 1x3 floating-point matrix (row vector)

set z {0+1i 2+3.5i 3.0+0i} ;# a complex vector of length 3
set u {{1 2 3}} ;# a 1x3 integer matrix (a row vector)
set v {{{1 2} {3 4}} {{5 6} {7 8}}} ;# a 2x2x2 integer tensor

set e {1.0 2 3} ;# a floating point vector of length 3
```

No external dependencies

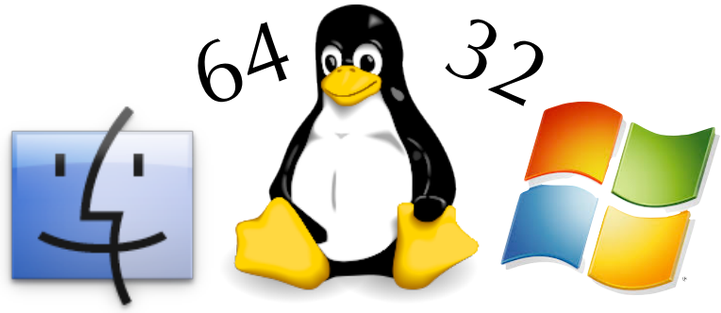


To compile VecTcl you need:

- a C compiler
- Tcl

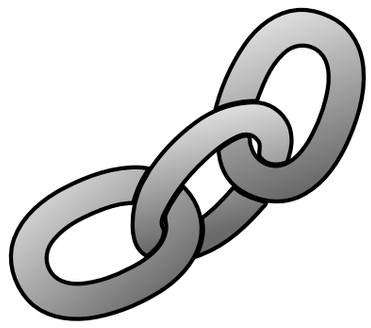
To run VecTcl you need:

- VecTcl
- TclOO

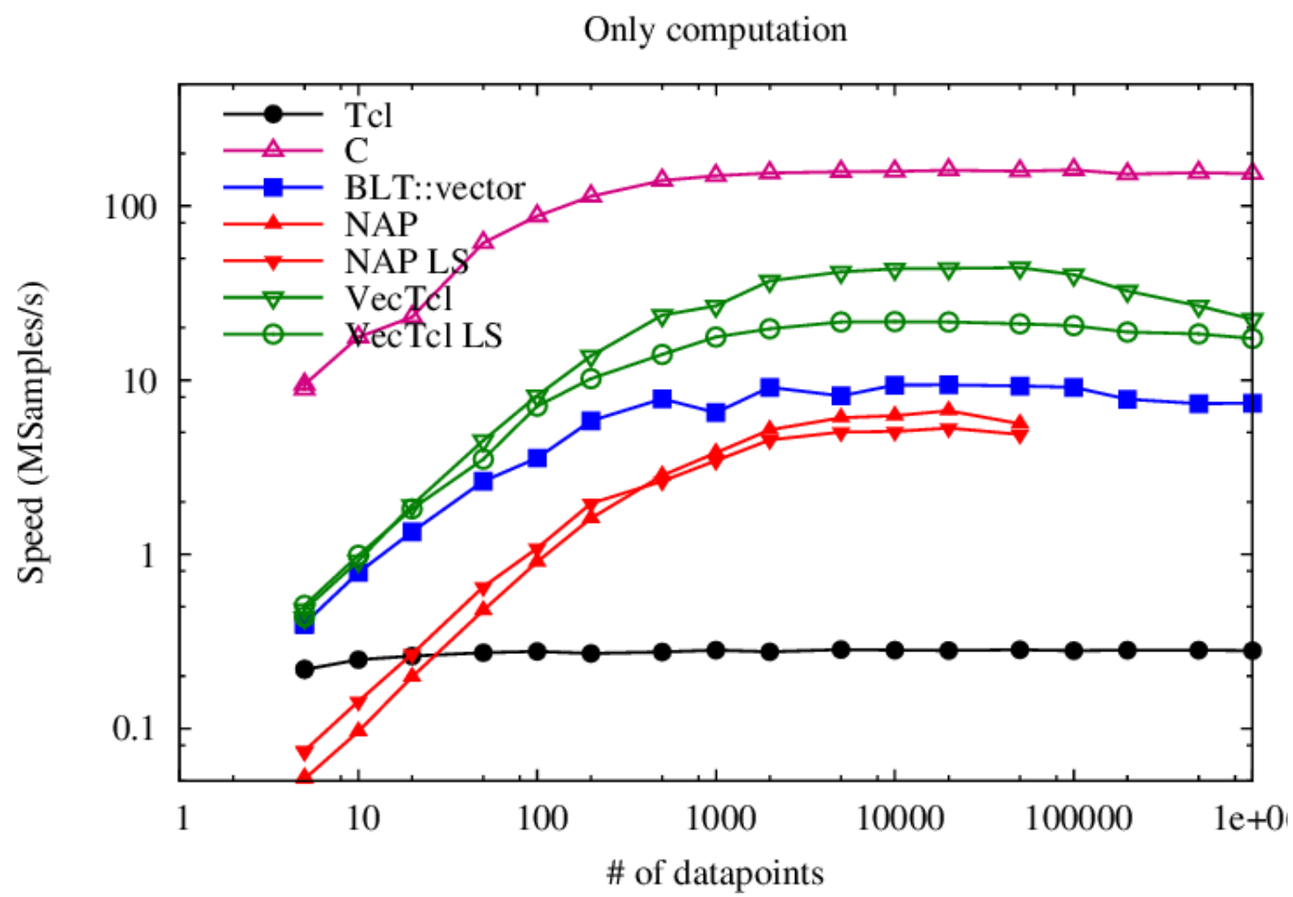


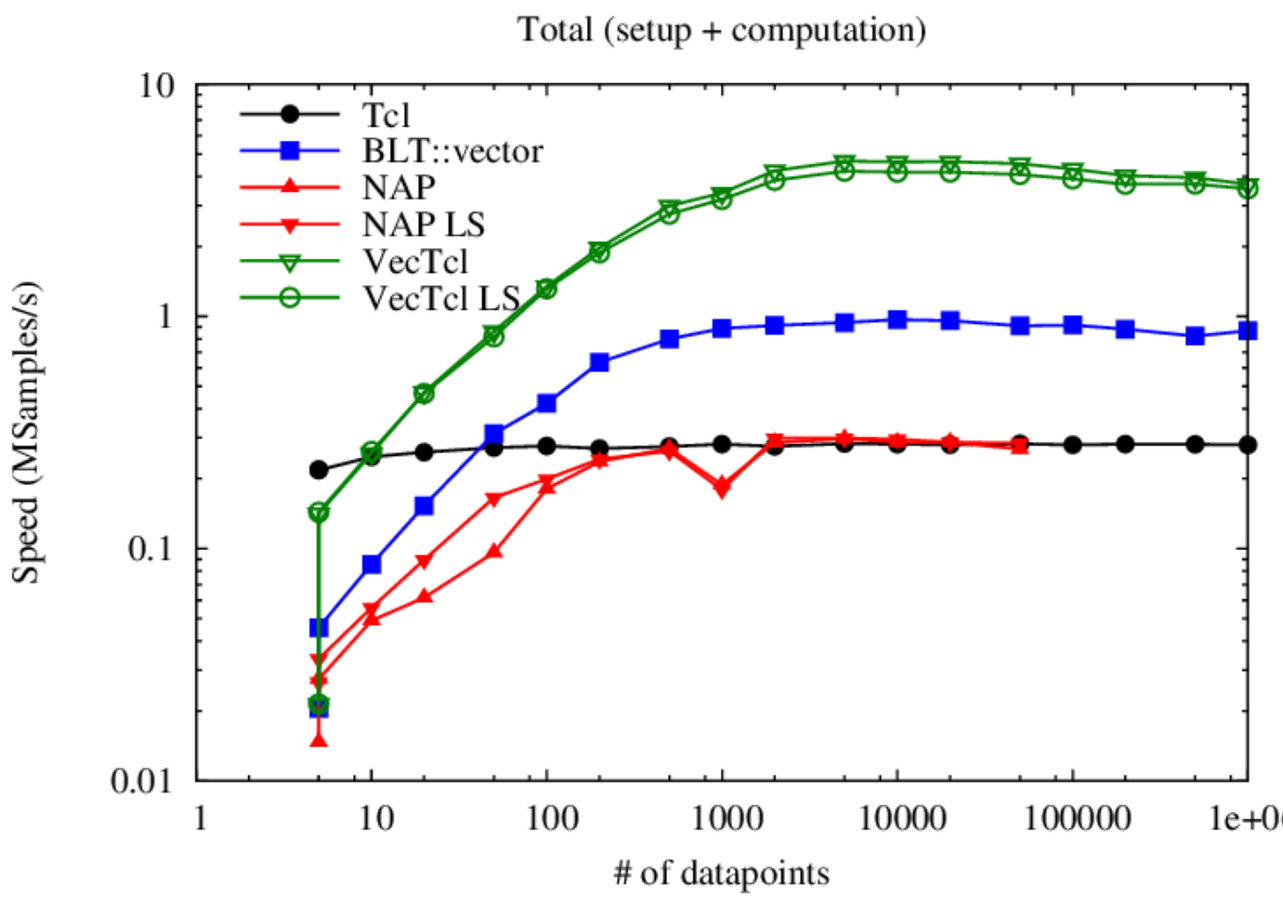
To rebuild VecTcl from scratch:

- autoconf
- tcllib::parsertools
- CLAPACK



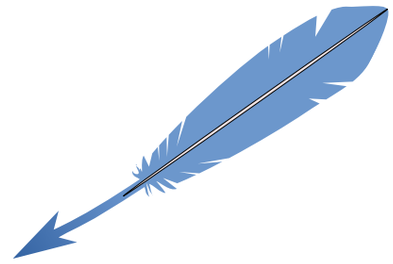
Benchmarks - linear regression





- VecTcl is 4x slower than C, but still faster than the others
- Shimmering is 5x slower than actual computation
- Competitors are still slower there

How does it work?



VecTcl is a 2-layered system

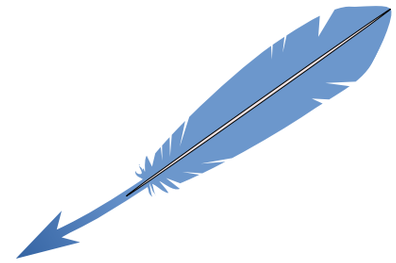
```
vexpr {
     a={1 2 3}
     c=2*(a+{4 5 6})
}
```

Compiler, written in Tcl

Runtime, written in C

```
proc numarray::compiledexpressionXX {} {
  upvar 1 a a
  upvar 1 c c
  set a {1 2 3}
  set c [numarray::* 2 [numarray::+ [set a] {4 5 6}]]
}
numarray::compiledexpressionXX
```

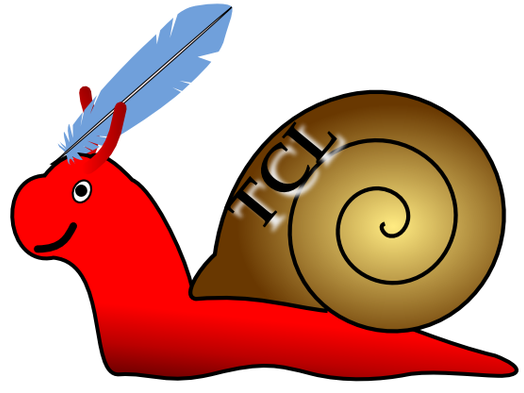
Why or when is it slow?



Tightly coded loops:

```
vexpr {
   a=zeros(1000);
   for i=0:999 {a[i]=2*i}
}
```

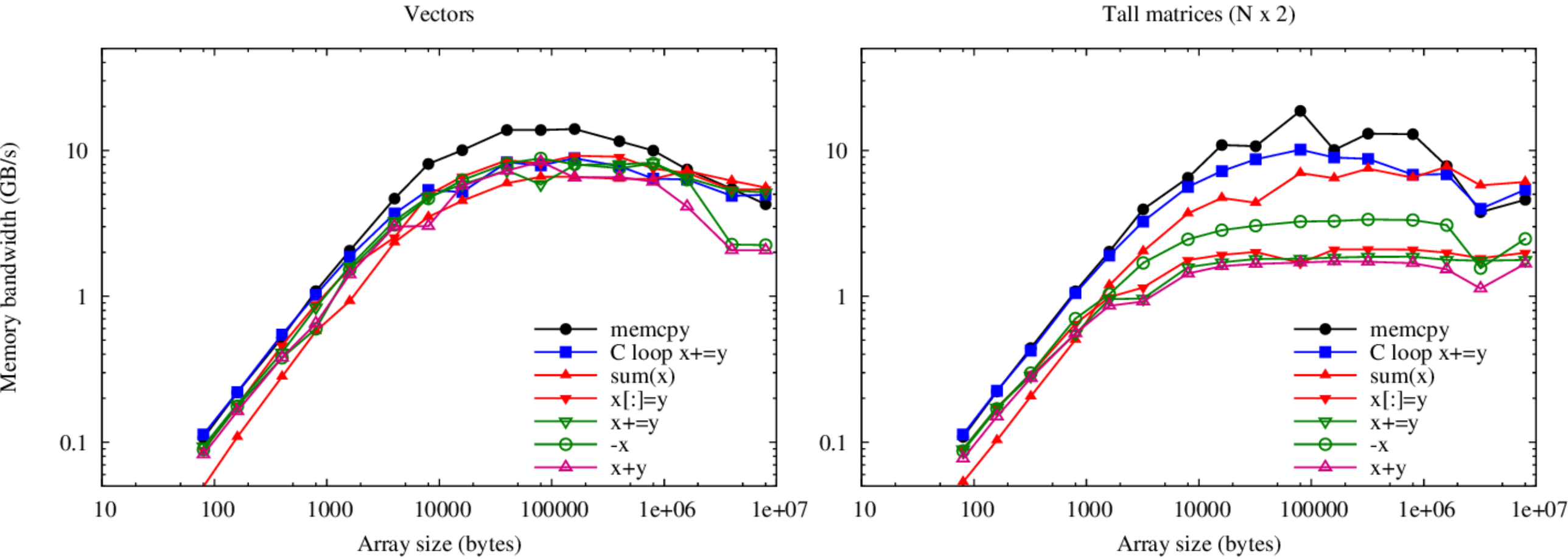
```
set a [zeros 1000]
set __temp1 999
for {set i 0} {$i <= $__temp1} {incr i 1} {
numarray::= a [list [list [set i] [set i] 1]]
[numarray::* 2 [set i]]
}
```



- Avoid if possible: `vexpr { a=2*linspace(0,999,1) }`
- Huge speed-up possible by JIT compilation (tcc4tcl?)

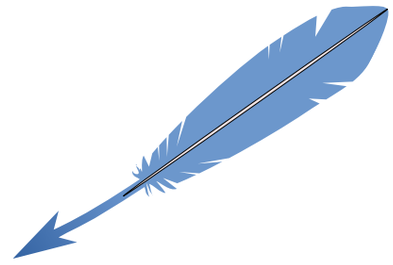
Why or when is it slow?

Speed of the elementary operations



- Vector operations close to the memory bandwidth
- Until ~10kbytes, the command dispatch dominates
- Matrix shape (currently) has a strong effect
- Improvement by OpenMP, BLAS, better iterators

Why or when is it slow?



Speed of complex operations

```
vexpr {
  r=a.*a + b.*b
}
```

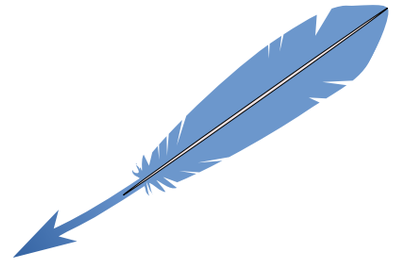
```
for (int i=0; i<N; i++) {
  r[i] = a[i]*a[i] + b[i]*b[i];
}
```

- 2*N* Flops
- 2*N* temporary storage
- 3 passes over the data

- 2*N* Flops
- 2 temporary registers
- 1 pass over the data

- Not easy to solve (short of JIT)
- More complex operations
- Blocking

Conclusion & The Future



- VecTcl provides an easy interface to numeric math in Tcl
- Value semantics (Tcl_Obj) in contrast to other packages
- Performance superior to other packages, but worse than C
- Performance could suffer by changes in Tcl 9
- JIT compilation possible using tcc4tcl